Graph Transformations – Worked Examples

Key Facts / Formulas

- **Reflections:** replacing y by $-y$ reflects a graph in the x-axis; replacing x by $-x$ reflects it in the y-axis
- **Translations:** $x \mapsto x - h$ shifts a graph h units right ($h > 0$) or left ($h < 0$); $y \mapsto y - k$ shifts it k units up or down
- **Dilations:** $x \mapsto \frac{x}{a}$ stretches horizontally by a factor a; $y \mapsto \frac{y}{b}$ stretches vertically by a factor b (enlargement if > 1, reduction if $0 < b < 1$)
- **Order matters:** applying dilations before translations usually gives a different image than the reverse
- Combined transformations can be described by tracing each change back to the basic graph.

Example 1 Reflection in the x-axis
Start with $y = \sqrt{x}$. Replace y by $-y \Rightarrow -y = \sqrt{x} \implies y = -\sqrt{x}$. Graph flips upside-down; domain $x \geq 0$, range $y \leq 0$.

$$\boxed{y = -\sqrt{x}}$$

Example 2 Horizontal translation
Graph $y = |x|$ is shifted 3 units right: $x \mapsto x - 3$. Equation becomes $y = |x - 3|$. Vertex moves from $(0, 0)$ to $(3, 0)$.

$$\boxed{y = |x - 3|}$$

Example 3 Vertical stretch
Begin with $y = \dfrac{1}{x}$. Apply vertical dilation factor 2: $y \mapsto \frac{y}{2}$ so $2y = \dfrac{1}{x} \Rightarrow y = \dfrac{1}{2x}$. Horizontal asymptote unchanged ($y = 0$); vertical asymptote $x = 0$.

$$\boxed{y = \frac{1}{2x}}$$

Example 4 Combined shift and stretch
Transform $y = x^2$ by "stretch vertically by $k = 3$ then up by 4". Equation: $y = 3x^2 + 4$. Vertex $(0, 0) \to (0, 4)$, axis $x = 0$.

$$\boxed{y = 3x^2 + 4}$$

Example 5 Horizontal dilation and reflection
Given $y = \sqrt{x}$, perform "reflect in y-axis then stretch horizontally by 5". Step1: reflection $x \mapsto -x$ gives $y = \sqrt{-x}$. Step2: dilation $x \mapsto \frac{x}{5} \Rightarrow y = \sqrt{-\frac{x}{5}}$. Domain $x \leq 0$, range $y \geq 0$.

$$\boxed{y = \sqrt{-\frac{x}{5}}}$$

Example 6 Identifying transformations
The graph of $y = -(x+2)^2 + 1$. Read off: shift left 2, reflect in x-axis (leading negative), shift up 1. Vertex at $(-2, 1)$, opens downward, axis $x = -2$.

$$\boxed{\text{Left2, reflect}x\text{-axis, up1}}$$

Example 7 Circle by translation
Start with unit circle $x^2 + y^2 = 1$. Translate centre to $(4, -3)$. Replace x by $x - 4$, y by $y + 3$: $(x-4)^2 + (y+3)^2 = 1$. Centre $(4, -3)$, radius 1.

$$\boxed{(x-4)^2 + (y+3)^2 = 1}$$

Example 8 Piecewise absolute value
Let $f(x) = |x - 2| + 1$. Describe transformations of $y = |x|$. Shift right 2, up 1. V-shape vertex at $(2, 1)$. Equation already given.

$$\boxed{\text{Right2, up1}}$$

Example 9 Order of transformations
Show two orders differ: start $y = x^3$, apply stretch $y \mapsto 2y$ and right shift $x \mapsto x - 1$. *OrderA:* stretch then shift $\Rightarrow y = 2(x-1)^3$. *OrderB:* shift then stretch $\Rightarrow y = 2x^3 - 2$. Distinct graphs, confirming order matters :contentReference[oaicite:10]index=10.

$$\boxed{\text{Order changes result}}$$

Example 10 Finding rule from graph
A transformed graph of $y = \dfrac{1}{x}$ has vertical asymptote $x = 3$, horizontal asymptote $y = -2$ and passes through $(4, -1)$. General form $y = \dfrac{k}{x-3} - 2$. Substitute point: $-1 = \dfrac{k}{4-3} - 2 \Rightarrow k = 1$.

$$\boxed{y = \frac{1}{x-3} - 2}$$