

Integral Calculus – Question Set

Basic antiderivatives

Q1 Find $\int (4x^5 - 3x^3 + 2x - 7) dx$.

Q2 Find $\int (5x^{-3} - \frac{2}{x} + 7) dx$.

Q3 Evaluate $\int (3e^{2x} - 4 \sin x + 5 \cos 3x) dx$.

Q4 Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$.

Q5 Evaluate $\int \left(\frac{6}{3x+2} \right) dx$.

Definite integrals

Q6 Compute $\int_1^3 (3x^2 - 4x + 5) dx$.

Q7 Evaluate $\int_0^\pi (\sin x + 1) dx$.

Q8 Compute $\int_{-2}^1 (x^3 - 2x) dx$.

Q9 Evaluate $\int_0^{\ln 5} e^x dx$.

Q10 Compute $\int_0^{\pi/2} \cos 2x dx$.

Substitution

Q11 $\int \frac{6x+4}{(3x+2)^2} dx$.

Q12 $\int x \cos(2x^2) dx$.

Q13 $\int \frac{2x}{x^2 + 5} dx.$

Q14 $\int \frac{1}{\sqrt{4 - 3x}} dx.$

Q15 $\int \frac{(2x - 1)}{(x^2 - x + 2)} dx.$

Integration by parts

Q16 $\int x e^{2x} dx.$

Q17 Show that $\int \ln x dx = x \ln x - x + C$ using parts.

Q18 $\int x \sin 3x dx.$

Q19 $\int x^2 e^{-x} dx.$

Q20 $\int (\ln x)^2 dx.$

Partial fractions

Q21 $\int \frac{3x + 5}{x^2 - x - 2} dx.$

Q22 $\int \frac{5x - 1}{(x + 2)(x - 1)} dx.$

Q23 $\int \frac{7}{x(x + 3)} dx.$

Q24 $\int \frac{2x + 3}{(x - 1)^2} dx.$

Q25 $\int_1^3 \frac{4}{x^2 - 4} dx.$

Exponentials and logarithms

Q26 $\int e^{3x} \ln x dx$ (use parts).

Q27 $\int \frac{\ln(2x)}{x} dx.$

Q28 $\int_1^e \ln x \, dx.$

Q29 $\int e^{-0.6x} \, dx.$

Q30 $\int \frac{1}{x \ln x} \, dx$ (assume $x > 1$).

Trigonometric integrals

Q31 $\int \sin 4x \, dx$ and $\int \cos 5x \, dx.$

Q32 $\int \sin x \cos x \, dx.$

Q33 $\int \sec^2 x \, dx.$

Q34 $\int \tan x \, dx$ (write $\tan x = \frac{\sin x}{\cos x}$).

Q35 $\int_0^{\pi/4} \frac{1}{\cos^2 x} \, dx.$

Areas and average values

Q36 Find the area between $y = x + 2$ and $y = x^2$ between their points of intersection.

Q37 Find the exact area between $y = 6x - x^2$ and $y = 2x$.

Q38 Find the average value of $f(x) = \sin x + 1$ on $[0, \pi]$.

Q39 Find the average value of $f(x) = x(6 - x)$ on $[0, 6]$.

Q40 The curve $y = 4x - x^2$ meets the x -axis at $x = 0$ and $x = 4$. Find the exact area enclosed.

Numerical integration

Q41 Use the trapezoidal rule with $n = 4$ to approximate $\int_0^2 e^{-x^2} \, dx.$

Q42 Apply the trapezoidal rule with $n = 6$ to approximate $\int_0^3 \frac{1}{1+x^2} dx$.

Q43 A velocity (m s^{-1}) is recorded each second for $t = 0, 1, \dots, 8$ as

t	0	1	2	3	4	5	6	7	8
v	0.0	2.3	4.1	5.0	5.4	5.1	4.4	3.2	2.0

Use the trapezoidal rule to estimate the displacement over the 8 seconds.

Mixed practice

Q44 $\int \frac{3x^2 - 4x + 1}{x^3} dx.$

Q45 $\int \frac{5x + 2}{x^2 + 4x + 5} dx.$

Q46 $\int \frac{dx}{x^2 + 9}.$

Q47 $\int_0^1 xe^{x^2} dx.$

Q48 $\int_{-1}^2 (|x| + 1) dx.$

Q49 If $F'(x) = \frac{1}{x^2 + 1}$ and $F(0) = 0$, find $F(1)$.

Q50 Show that $\frac{d}{dx} \left(\frac{1}{2} \ln(1 + x^2) \right) = \frac{x}{1 + x^2}$, hence evaluate $\int \frac{x}{1 + x^2} dx$.

Challenge Question

Q51 Area bounded by a line and a parabola with parameter

For $k > 0$, consider the region enclosed by the curves $y = kx$ and $y = 4x - x^2$.

- Find the x -coordinates of the intersection points in terms of k .
- For which values of k does a bounded region exist?
- Express the enclosed area $A(k)$ as a single definite integral and simplify it to an explicit function of k .
- Determine the value of k that maximises $A(k)$, and find this maximum area.