Differential Calculus – Question Set

Basics

Q1 Differentiate $f(x) = 4x^5 - 3x^3 + 2x - 7$.

Q2 Find $f'(x)$ for $f(x) = \dfrac{5}{\sqrt{x}} - 3x^{-2} + 7$.

Q3 Differentiate $y = (x^2 + 3x - 1)^4$.

Q4 Find the derivative of $y = \sqrt{9 - 4x}$.

Q5 If $f(x) = x^3 - 4x$, find $f'(2)$ and the equation of the tangent at $x = 2$.

Product and Quotient Rules

Q6 Differentiate $y = (2x^2 - 5)e^{3x}$.

Q7 Differentiate $y = (x^2 + 1)\sin x$.

Q8 Differentiate $y = \dfrac{3x^2 - 4x + 1}{x^3}$ and simplify.

Q9 Find $\dfrac{dy}{dx}$ for $y = \dfrac{\sin x}{x}$.

Q10 Differentiate $y = \dfrac{(x-1)^2}{(x+1)^3}$.

Chain Rule

Q11 Differentiate $y = \big(\sin 4x\big)^5$.

Q12 Find $\dfrac{dy}{dx}$ for $y = \cos^3(2x - 1)$.

Q13 Differentiate $y = \ln(5 - 2x - 3x^2)$.

Q14 Differentiate $y = e^{(x^2 - 3x)}$.

Q15 Differentiate $y = \sqrt{(2x - 1)^3}$.

Exponential and Logarithmic

Q16 Differentiate $y = e^{2x} \ln(x)$.

Q17 Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{x^2+4}{\sqrt{x}}\right)$.

Q18 Differentiate $y = 7^x \cdot e^{-3x}$.

Q19 Differentiate $y = \ln(x^2+1)$ and hence find the tangent at $x = 2$.

Trigonometric Derivatives

Q20 Differentiate $y = \tan x + \sec x$.

Q21 Find $\frac{dy}{dx}$ for $y = \sin(3x)\cos(2x)$.

Q22 Differentiate $y = \tan^2(5x-2)$.

Q23 Show that $\frac{d}{dx}(\cot x) = -\csc^2 x$ using the quotient rule.

Implicit Differentiation

Q24 For $x^2 + xy + y^2 = 7$, find $\frac{dy}{dx}$ in terms of x, y.

Q25 Given $\sin(x+y) = xy$, find $\frac{dy}{dx}$.

Q26 For the curve $x^3 + y^3 = 6xy$, find the slope at the point $(2, 2)$.

Q27 The curve $x^2 - 3xy + y^2 = 1$ passes through $(1, 1)$. Find the equation of the tangent there.

Parametric

Q28 $x = t^2 + 1$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ and $\left.\frac{d^2y}{dx^2}\right|_{t=1}$.

Q29 $x = \sin t,\ y = \cos t$. Find $\frac{dy}{dx}$ and the slope at $t = \frac{\pi}{4}$.

Q30 $x = e^t$, $y = te^t$. Find $\frac{dy}{dx}$ and the equation of the tangent at $t = 0$.

Second Derivatives and Classification

Q31 For $f(x) = x^4 - 4x^2 + 3$, find $f'(x)$, $f''(x)$ and classify the stationary point at $x = 0$.

Q32 Find the stationary points of $y = x^3 - 6x^2 + 9x + 4$ and classify each using y''.

Q33 Determine points of inflection (if any) for $y = x^3 - 3x$.

Tangents and Normals

Q34 Find the tangent and normal to $y = \ln(x^2 + 1)$ at $x = 2$.

Q35 For $y = \dfrac{x+1}{x-2}$, find the tangent and normal at $x = 3$.

Q36 A curve passes through $(1, 3)$ and has derivative $y' = 2x + \dfrac{1}{2\sqrt{x}}$. Find the tangent at $x = 1$.

Logarithmic Differentiation

Q37 Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = \dfrac{(x^2+1)^5\sqrt{3x-1}}{e^{2x}}$.

Q38 Differentiate $y = (x^2 + 3x + 2)^x$.

Mixed Practice

Q39 If $y = \dfrac{x}{x^2+1}$, find all stationary points and classify them.

Q40 For $y = x^2 e^{-x}$, find the x value of the maximum and the maximum y value.

Q41 Find $\dfrac{dy}{dx}$ for $y = \arctan\left(\sqrt{x}\right)$.

Q42 For $y = \ln(\sin x)$ with $0 < x < \pi$, find y' and y''.

Q43 Show that if $y = x^{\sin x}$ then $\dfrac{y'}{y} = \dfrac{\sin x}{x} + \cos x \ln x$.

Challenge Question

Q44 Envelope and common tangent

Consider the family of curves $y = \dfrac{k}{x} + x$ where $k > 0$.

(a) For fixed k, find the stationary point of y and classify it.

(b) A straight line $y = mx + c$ is tangent to one member of the family at $x = a$. Show that $m = 1 - \dfrac{k}{a^2}$ and $c = \dfrac{2k}{a} + a$.

(c) The *envelope* of the family is the curve tangent to every member at some point. By eliminating k between $y = \dfrac{k}{x} + x$ and $y' = 1 - \dfrac{k}{x^2}$, find the equation of the envelope and hence its point of contact with the kth curve.

(d) Prove that each member of the family has exactly one common tangent with the envelope, and find the gradient of that common tangent in terms of k.