

Area, Surface Area Volume – Worked Examples

Formulas & Key Theory

- **2D Shape Area Formulas** Rectangle $A = lw$; triangle $A = \frac{1}{2}bh$; trapezium $A = \frac{1}{2}(a + b)h$.
Circle $A = \pi r^2$; sector $A = \frac{\theta}{360^\circ}\pi r^2$.
- **Surface Area Formulas** Right prism $SA = 2A_{\text{base}} + P_{\text{base}}h$. Cylinder $SA = 2\pi r^2 + 2\pi rh$.
Cone $SA = \pi r^2 + \pi r\ell$ (slant height ℓ). Pyramid $SA = A_{\text{base}} + \frac{1}{2}P_{\text{base}}\ell$.
- **Volumes** Prism $V = A_{\text{base}}h$; cylinder $V = \pi r^2h$. Pyramid $V = \frac{1}{3}A_{\text{base}}h$; cone $V = \frac{1}{3}\pi r^2h$.
Sphere $V = \frac{4}{3}\pi r^3$.
- **Composite Solids** – split into simple parts, find individual volumes or areas, then add/subtract.
- **Unit conversions** 1 cm = 10 mm; 1 cm² = 100 mm²; 1 m³ = 1 000 000 cm³.

Example 1 Composite Area (L-shape)

A courtyard is an L-shape made from two rectangles: 6 m $\times$ 4 m and 3 m $\times$ 2 m.

$$A = 6 \times 4 + 3 \times 2 = 24 + 6 = \boxed{30 \text{ m}^2}$$

Example 2 Sector of a Circle

Find the area of a sector with radius 9 cm and angle 60°.

$$A = \frac{60^\circ}{360^\circ}\pi r^2 = \frac{1}{6}\pi(81) = 13.5\pi \approx \boxed{42.4 \text{ cm}^2}$$

Example 3 Surface Area of a Rectangular Prism

A box 12 cm $\times$ 8 cm $\times$ 5 cm.

$$SA = 2(lw + lh + wh) = 2(96 + 60 + 40) = \boxed{392 \text{ cm}^2}$$

Example 4 Surface Area of a Closed Cylinder

Radius 7 cm, height 15 cm.

$$SA = 2\pi r^2 + 2\pi rh = 2\pi(49) + 2\pi(7)(15) = 98\pi + 210\pi = 308\pi \approx \boxed{968 \text{ cm}^2}$$

Example 5 Surface Area of a Right Cone

Radius 5 cm; slant height $\ell = 13$ cm.

$$SA = \pi r^2 + \pi r \ell = 25\pi + 65\pi = 90\pi \approx \boxed{283 \text{ cm}^2}$$

Example 6 Volume of a Triangular Prism

Base triangle $b = 6$ m, $h = 4$ m; prism length 10 m.

$$A_{\triangle} = \frac{1}{2}(6)(4) = 12 \text{ m}^2 \quad \Rightarrow \quad V = 12 \times 10 = \boxed{120 \text{ m}^3}$$

Example 7 Volume of a Cylinder

Water tank radius 1.2 m, height 2.5 m.

$$V = \pi r^2 h = \pi(1.44)(2.5) = 3.6\pi \approx \boxed{11.3 \text{ m}^3}$$

Example 8 Volume of a Cone

Ice-cream cone radius 3 cm, height 10 cm.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(9)(10) = 30\pi \approx \boxed{94.2 \text{ cm}^3}$$

Example 9 Volume of a Sphere

Ball radius 8 cm.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(512) = \frac{2048\pi}{3} \approx 2.15 \times 10^3 \text{ cm}^3$$

$$\boxed{2.15 \text{ L (since } 1000 \text{ cm}^3 = 1 \text{ L)}}$$

Example 10 Composite Solid

A cylindrical silo (radius 4 m, height 12 m) sits on a hemispherical base of the same radius.

$$V_{\text{cyl}} = \pi r^2 h = \pi(16)(12) = 192\pi$$

$$V_{\text{hemi}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(64) = \frac{128}{3}\pi$$

$$V_{\text{total}} = 192\pi + \frac{128}{3}\pi = \frac{704}{3}\pi \approx 737 \text{ m}^3$$

$$\boxed{7.37 \times 10^2 \text{ m}^3}$$