Exponential & Logarithmic Functions – Question Set

Rewrite & Evaluate

Q1 (a) Rewrite $3^x = 17$ in logarithmic form (b) Rewrite $y = \log_2 9$ in exponential form

Q2 Evaluate to three decimal places: (a) $\log_5 42$ (b) $\ln 0.35$

Q3 Without a calculator, simplify:

(a) $\log_{10} 1000$

(b) $\log_3 27^{\frac{2}{3}}$

(c) $\ln e^{-4}$

Q4 Order the following from least to greatest: $\log_2 6,\ \log_5 6,\ \log_{10} 6$.

Log Laws & Simplification

Q5 Use log laws to expand: $\log_a\big(12x^3\sqrt{y}\big)$.

Q6 Simplify: $\dfrac{1}{3}\log_2 16 + \log_2 5 - \log_2 10$.

Q7 Show that $\log_b p + \log_b q - \log_b r = \log_b\Big(\dfrac{pq}{r}\Big)$.

Solving Exponential / Log Equations

Q8 Solve for x: $4^x = 96$ (exact form then 2 d.p.).

Q9 Solve: $\ln(3x - 5) = 2$.

Q10 Solve: $2^{2x-1} = 5^x$ (2 d.p.).

Q11 Find t if $7e^{0.15t} = 88$.

Exponential Growth & Decay

Q12 A culture grows from 9.0×10^4 to 1.17×10^5 in 2 h. Assuming $N = N_0 b^t$, find b and predict the count after 6 h.

Q13 A radioactive isotope decays by 12 % each hour.

(a) Write the decay factor b.

(b) Determine the half-life (nearest minute).

Q14 A bank compounds interest monthly at 4.2 % p.a. How long (years, 1 d.p.) to double an investment?

Graphs & Features

Q15 Sketch $y = 5(1.3)^x$, indicating intercept and asymptote.

Q16 State the axis of symmetry and vertex for $y = -x^2 + 6x - 4$.

Q17 For $y = \dfrac{8}{x+3} - 1$, give the equations of both asymptotes and the y-intercept.

Q18 Explain, without plotting points, whether $y = 0.4^x$ is increasing or decreasing.

Modelling & Interpretation

Q19 Energy output of a solar cell is modelled by $E = 140e^{-0.07t}$ W, t hours after sunset.

(a) What is the initial output?

(b) When will the output drop to 40 W?

Q20 Ticket sales follow $S = 4200\big(1 - e^{-0.3d}\big)$ with d days since release. Find sales after 5 days and the limiting total sales.

Q21 A medicine's concentration halves every 90 min. Write an equation for concentration C if the initial dose is 220 mg, then find C after 4 h.

Mixed Practice

Q22 (a) Use change of base to show $\log_7 27 = \dfrac{\ln 27}{\ln 7}$ (b) Hence evaluate $\log_7 27$ to 3 d.p.

Q23 Compare gradients at $x = 0$ for $y = 2^x$ and $y = e^x$; which rises faster initially?

Q24 Show that $\log_3 x = \dfrac{\ln x}{\ln 3}$ implies $\dfrac{d}{dx}\big(\log_3 x\big) = \dfrac{1}{x \ln 3}$.

Challenge Question

Q25 Logistic Growth Model A fish population P (in thousands) in a dam follows

$$P(t) = \frac{80}{1 + 15e^{-0.45t}}, \qquad t \geq 0 \text{ years.}$$

(a) Show that $P(0) = 5$ (thousand).

(b) Determine the limiting population as $t \to \infty$.

(c) After how many years will the population reach 60 000 fish?

(d) Compute $P'(t)$ and interpret the sign of P' when $t = 0$ and when $t \to \infty$.