Trigonometry – Worked Examples

Example 1 Angle of Elevation

A surveyor stands 45m from the base of a tower. The angle of elevation to the top is 37°. Find the height of the tower.

$$\tan 37^\circ = \frac{h}{45} \quad \Rightarrow \quad h = 45 \tan 37^\circ \approx 34.0 \text{ m}$$

$$\boxed{h \approx 34.0 \text{ m}}$$

Example 2 Angle of Depression

From a lighthouse 28m above sea level, the angle of depression to a boat is 12°. How far is the boat from the base of the lighthouse?

$$\tan 12^\circ = \frac{28}{d} \quad \Rightarrow \quad d = \frac{28}{\tan 12^\circ} \approx 131.6 \text{ m}$$

$$\boxed{d \approx 132 \text{ m (to nearest m)}}$$

Example 3 True Bearing

Point B lies 4km from A on a true bearing of 065°. Find the displacements in the north and east direction from A to B.

$$\text{Easting} = 4 \sin 65^\circ \approx 3.63 \text{ km}, \quad \text{Northing} = 4 \cos 65^\circ \approx 1.69 \text{ km}$$

$$\boxed{E = 3.63 \text{ km},\ N = 1.69 \text{ km}}$$

Example 4 Compass Bearing Conversion

A hiker walks on a compass bearing $S30^\circ E$. Write the equivalent true bearing.

Measured clockwise from North: $180^\circ + 30^\circ = 210^\circ$.

$$\boxed{210^\circ}$$

Example 5 3-D Pythagoras

A rectangular prism has edges 5cm, 8cm and 12cm. Find the length of the space diagonal.

$$d = \sqrt{5^2 + 8^2 + 12^2} = \sqrt{25 + 64 + 144} = \sqrt{233} \approx 15.3 \text{ cm}$$

$$\boxed{d \approx 15.3 \text{ cm}}$$

Example 6 3-D Trigonometry with Elevation

A drone flies directly above point P at a height of 50m. An observer at point Q, 120m east and 40m north of P, measures the angle of elevation to the drone. Find this angle.

Horizontal distance $d = \sqrt{120^2 + 40^2} = \sqrt{16\,000} = 126.5$ m.

$$\tan\theta = \frac{50}{126.5} \Rightarrow \theta = \arctan\big(0.395\big) \approx 21.6^\circ$$

$$\boxed{\theta \approx 22^\circ}$$

Example 7 Sine Rule – Unknown Side

In $\triangle ABC$, $A = 42^\circ$, $B = 75^\circ$, $a = 6.4$ cm. Find side b.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \Rightarrow \quad b = \frac{6.4\sin 75^\circ}{\sin 42^\circ} \approx 9.3 \text{ cm}$$

$$\boxed{b \approx 9.3 \text{ cm}}$$

Example 8 Cosine Rule – Unknown Angle

For $\triangle PQR$, $PQ = 7$ cm, $PR = 9$ cm, $QR = 11$ cm. Find $\angle P$.

$$\cos P = \frac{7^2 + 9^2 - 11^2}{2 \cdot 7 \cdot 9} = \frac{49 + 81 - 121}{126} = \frac{9}{126} = 0.0714$$

$$P = \arccos(0.0714) \approx 85.9^\circ \quad \Longrightarrow \quad \boxed{\angle P \approx 86^\circ}$$

Example 9 Area Rule

$\triangle XYZ$ has $XY = 12$ m, $YZ = 15$ m and $\angle Y = 58^\circ$. Find the area.

$$A = \frac{1}{2}(12)(15)\sin 58^\circ \approx 90\,(0.848) \approx 76.3 \text{ m}^2$$

$$\boxed{A \approx 76 \text{ m}^2}$$

Example 10 Unit-Circle Identity

Verify $\sin(180^\circ - \alpha) = \sin\alpha$ for $\alpha = 40^\circ$.

$$\sin(180^\circ - 40^\circ) = \sin 140^\circ \approx 0.643, \quad \sin 40^\circ \approx 0.643$$

$$\boxed{\text{Equal, identity confirmed}}$$

Example 11 Exact Trig Values

Find $\sin 45^\circ$, $\cos 30^\circ$, $\tan 60^\circ$.

$$\sin 45^\circ = \frac{\sqrt{2}}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 60^\circ = \sqrt{3}$$

$$\boxed{\frac{\sqrt{2}}{2},\ \frac{\sqrt{3}}{2},\ \sqrt{3}}$$

Example 12 Trig Equation with Supplementary Angle

Solve $\sin\theta = 0.5$ for $0^\circ \leq \theta \leq 180^\circ$.

$$\theta = 30^\circ \quad \text{or} \quad \theta = 180^\circ - 30^\circ = 150^\circ$$

$$\boxed{\theta = 30^\circ,\ 150^\circ}$$

Example 13 Ambiguous Case – Sine Rule

In $\triangle ABC$, $a = 9$ cm, $b = 11$ cm, $A = 35^\circ$. Find possible values of angle B.

$$\frac{\sin B}{b} = \frac{\sin 35^\circ}{9} \quad \Rightarrow \quad \sin B = \frac{11\sin 35^\circ}{9} \approx 0.701$$

$B_1 = 44.5^\circ$. Supplement gives $B_2 = 180^\circ - 44.5^\circ = 135.5^\circ$ (valid as $B_2 + A < 180^\circ$).

$$\boxed{B \approx 44.5^\circ \text{ or } 135.5^\circ}$$