Further Graph Transformations and Modelling – Question Set

Reading and producing transformations

Q1 The parent is $f(x) = \dfrac{1}{x}$. For $g(x) = -3\,f(2(x-4)) + 5$, state the vertical and horizontal asymptotes, and find the x- and y-intercepts.

Q2 For $f(x) = \sqrt{x}$, define $g(x) = 2\sqrt{1-3x} - 4$. State the domain and range of g.

Q3 Sketch $y = |2x-5| - 3$. Give the vertex and all intercepts.

Q4 Write the transformation in the form $y = A\,f(B(x-h)) + k$ that maps $y = \sqrt{x}$ to $y = -\sqrt{3(x+2)} + 1$.

Q5 The curve $y = \dfrac{p}{x-q} + r$ has asymptotes $x = 3$ and $y = -2$ and passes through $(1, 1)$. Find p, q, r.

Q6 A graph of the form $y = A\dfrac{1}{B(x-h)} + k$ has asymptotes $x = 2$ and $y = 1$ and passes through $\left(4, \frac{3}{2}\right)$. Find one possible pair (A, B) and write the function.

Q7 The function $y = \dfrac{a}{(x-1)^2} + c$ has a minimum value 5 and passes through $(3, 6)$. Find a, c.

Asymptotes, intercepts, domains, ranges

Q8 For $y = \dfrac{-4}{x+5} + 7$, find: vertical asymptote, horizontal asymptote, intercepts.

Q9 For $y = \dfrac{3}{(x-2)^2} - 5$, find the range and the y-intercept.

Q10 Consider $y = 4 - \sqrt{5-2x}$. State the domain.
Find all intercepts.

Q11 For $y = \ln(3-2x) + 1$, state the domain, vertical asymptote and x-intercept (exact).

Q12 If $y = 2|x+1| - 5$, write the piecewise form and sketch key points.

Determine parameters from conditions

Q13 Find a, b so that $y = a\sqrt{b(x-1)} + 2$ passes through $(5, 6)$ and $(2, 2)$.

Q14 The reciprocal model $y = \dfrac{A}{x-1} + B$ has x-intercept $x = 5$ and y-intercept $y = 3$. Find A, B.

Q15 A transformed cubic $y = A(x-h)^3 + k$ passes through $(1, 2)$, has a stationary point at $(1, 2)$, and also passes through $(3, 18)$. Find A, h, k.

Q16 A logistic-type reciprocal $y = \dfrac{L}{1 + ae^{-kx}}$ has horizontal asymptote $y = L = 120$ and passes through $(0, 40)$ and $(2, 80)$. Find a and k.

Model fitting: exponential, power, logarithmic

Q17 Assume $y = ab^x$ fits $(0, 4)$ and $(3, 27)$. Find a, b and predict $y(5)$.

Q18 Bacterial mass follows $M = Ke^{rt}$. Data: (t, M) in hours/grams: $(0, 2.4)$, $(5, 6.7)$. Find K, r and predict at $t = 9$.

Q19 Drag force F varies with speed v as a power: $F = kv^n$. Measurements: $(v, F) = (2, 18), (5, 140)$. Find k and n (2 d.p.).

Q20 Timber beam deflection d (mm) varies with load W (kN) by $d = a + b \ln W$. Using $(W, d) = (1.0, 4.2)$ and $(e, 6.0)$, find a, b and predict d at $W = 4.0$.

Q21 A dataset is suspected to follow $y = \dfrac{p}{x-q} + r$. Given points $(2, 10)$, $(6, 2)$ and the horizontal asymptote $y = 1$, determine p, q, r.

Piecewise and absolute-value reasoning

Q22 Write $y = |x^2 - 4x|$ as a piecewise function by solving where $x^2 - 4x \geq 0$. Hence find all x-intercepts.

Q23 Solve $|2x - 5| - 3 \leq 1$ and express the solution set.

Q24 The graph $y = |mx + c| + k$ has vertex at $(2, -3)$ and passes through $(0, 1)$. Find m, c, k.

Transforming restricted domains

Q25 $f(x) = \sqrt{4 - x}$ has domain $(-\infty, 4]$. For $g(x) = -\dfrac{1}{2} f\left(\dfrac{x+1}{2}\right) + 3$, find the domain and range of g.

Q26 If f has range $[2, \infty)$ and we define $h(x) = -3f(2 - x) + 5$, what is the range of h?

Q27 A rational $f(x) = \dfrac{1}{x-3}$ is restricted to $x > 5$. For $g(x) = 2f(3-2x) - 1$, find the domain for g induced by the restriction on f.

Short application modelling

Q28 Cooling of a drink: temperature T (°C) follows $T(t) = T_{\text{room}} - (T_{\text{room}} - T_0)e^{-kt}$. If $T_{\text{room}} = 22$, $T_0 = 80$, and $T(10) = 40$, find k and predict $T(20)$.

Q29 Viral reach $R(t)$ satisfies $R(t) = \dfrac{L}{1 + ae^{-kt}}$ with $L = 100{,}000$. If $R(0) = 5{,}000$ and $R(3) = 30{,}000$, find a, k and estimate $R(6)$.

Q30 A pollutant concentration follows $C(t) = \dfrac{A}{t+q} + C_\infty$. Given $C(0) = 42$, $C(5) = 18$, and long-term level $C_\infty = 6$, find A, q.

Challenge Question

Q31 **Selecting a model and linearising**

A device is tested at different inputs x with outputs y measured as:

x	1	2	3	5	8	12
y	4.1	5.7	6.6	7.8	8.9	9.6

Engineers suspect either a logarithmic model $y = a + b\ln x$ or a reciprocal-approach model $y = c - \dfrac{d}{x}$.

(a) Show how each model can be linearised for regression (state the transformed variables).

(b) Using pairs of points $(1, 4.1)$ and $(12, 9.6)$ together with $(3, 6.6)$ as a check, quickly estimate parameters for both models by hand.

(c) For each model, use your parameters to predict y at $x = 20$. Which model gives the smaller absolute error against the heuristic trend $y \approx 10$ as x grows?

(d) Explain, using asymptotes/range behaviour, which model is more physically plausible if the device output is known to have an upper bound of about 10.