Differentiation – Question Set

First Principles & Power Rule

Q1 Use first principles to find $f'(x)$ for $f(x) = x^2 + 3$.

Q2 Show from first principles that $\frac{d}{dx}(x^3) = 3x^2$.

Q3 (a) Differentiate $y = 7x^4$ (b) $y = 5x^{-2}$

Q4 Find $\frac{d}{dx}(4\sqrt{x})$.

Q5 If $y = \frac{3}{\sqrt[3]{x}}$, determine y'.

Sum, Difference & Constant Multiple

Q6 (a) $y = 6x^3 - 5x$ (b) $y = 9 - 4x^2 + x^5$

Q7 Differentiate $y = 2x^{\frac{1}{2}} + 7x^{\frac{3}{2}} - 8$.

Q8 Given $f(x) = x^4 - 2x^2 + 1$ and $g(x) = 5x - 3$, find $\frac{d}{dx}[3f(x) - 2g(x)]$.

Tangents & Normals

Q9 For $y = x^3 - 2x$, find the gradient at $x = -1$.

Q10 Determine the equation of the tangent to $y = 4x^2 - 7$ at $x = 1.5$.

Q11 A curve has equation $y = x^4 - 16x$. Find the equation of the normal at $x = 2$.

Q12 A function satisfies $f(1) = 6$ and $f'(1) = -3$. Write the tangent line through $(1, 6)$.

Stationary Points & Shape

Q13 Find and classify the stationary points of $y = 2x^3 - 9x^2 + 12x$.

Q14 Determine the x-coordinates of any local maxima of $y = x^4 - 6x^2 + 4$.

Q15 (a) Locate stationary points for $y = 5x^2 - 20x + 17$ (b) Is the point a max or min?

Q16 If $f'(x) = 0$ at $x = 4$ and $f''(4) = 0$, what further test is required to classify the point?

Motion – Velocity & Acceleration

Q17 A particle moves so that $s(t) = 3t^3 - 12t$ (metres).

- **(a)** Find $v(t)$ and $a(t)$.
- **(b)** When is the particle momentarily at rest?

Q18 Given $s(t) = 10\sqrt{t} - t^2$, find velocity at $t = 4\,\text{s}$.

Q19 A car's velocity is $v(t) = 12t^2 - 30t + 18$ (m/s). Determine its acceleration after 2.5s.

Optimisation Basics

Q20 The area of a rectangle is $A = x(30 - 2x)\,\text{m}^2$.

- **(a)** Find $A'(x)$.
- **(b)** What width x maximises the area?

Q21 A manufacturer wants to minimise surface area of an open box with square base and fixed volume $4900\,\text{cm}^3$. List the derivative equation that must be solved to find the optimal base edge x.

Q22 Show that among all right triangles with hypotenuse 10cm, the isosceles right triangle encloses the maximum area.

Mixed Practice

Q23 (a) Differentiate $y = (5x + 2)(x^2 - 1)$ (b) Differentiate $y = \dfrac{6}{x^3} + 4x$

Q24 Find the gradient of $y = x^{\frac{4}{3}}$ at $x = 1$.

Q25 Prove that $\dfrac{d}{dx}(x^n) = nx^{n-1}$ for $n = -1$ using the limit definition.

Q26 The line $y = 3x - 8$ is tangent to the curve $y = x^3 + ax^2 + bx$. Find a and b.

Challenge

Q27 **Designing a Minimum-Cost Can** A cylindrical can must hold 1.2L. Metal for the side costs \$0.04/cm^2 and for the circular top/bottom \$0.06/cm^2.

- **(a)** Express the total cost $C(r)$ in terms of the radius r (cm).
- **(b)** Show that $C'(r) = 0$ leads to $r^3 = \dfrac{0.60}{\pi}$ (approx.).
- **(c)** Hence find the optimal radius and height, and the minimum cost (nearest cent).