Differentiation – Answer Sheet

Q1 $f'(x) = 2x$

Q2 $\dfrac{d}{dx}(x^3) = 3x^2$

Q3 (a) $28x^3$ (b) $-\dfrac{10}{x^3}$

Q4 $\dfrac{2}{\sqrt{x}}$

Q5 $y' = -x^{-4/3} = -\dfrac{1}{x^{4/3}}$

Q6 (a) $18x^2 - 5$ (b) $-8x + 5x^4$

Q7 $x^{-1/2} + 10.5x^{1/2} = \dfrac{1}{\sqrt{x}} + 10.5\sqrt{x}$

Q8 $12x^3 - 12x - 10$

Q9 1

Q10 $y = 12x - 16$

Q11 $y = -\dfrac{1}{16}x - \dfrac{127}{8}$

Q12 $y = -3x + 9$

Q13 $x = 1$ (max, $y = 5$), $x = 2$ (min, $y = 4$)

Q14 Local maxima at $x = 0$

Q15 (a) $x = 2,\ y = -3$(b) minimum

Q16 Use higher–order or first-derivative sign test around $x = 4$

Q17 (a) $v = 9t^2 - 12,\ a = 18t$ (b) Rest at $t = \sqrt{\frac{4}{3}} \approx 1.16$s

Q18 $v(4) = 5/2 - 8 = -5.5\,\mathrm{m\ s^{-1}}$

Q19 $a(2.5) = 30\,\mathrm{m\ s^{-2}}$

Q20 (a) $A'(x) = 30 - 4x$(b) $x = 7.5$m

Q21 Solve $S'(x) = 2x - \dfrac{9800}{x^2} = 0$

Q22 Max area when legs $= 5\sqrt{2}$ cm (isosceles)

Q23 (a) $15x^2 + 4x - 5$(b) $-18/x^4 + 4$

Q24 Gradient $= \dfrac{4}{3}$

Q25 $\dfrac{d}{dx}\left(x^{-1}\right) = -x^{-2}$

Q26 $a = -2,\ b = -1$

Challenge — Worked Solution

(a) Cost function

Volume: $\pi r^2 h = 1200\,\mathrm{cm}^3$ $(1.2\,\mathrm{L}) \Rightarrow h = \dfrac{1200}{\pi r^2}$

Side area $= 2\pi rh$ at \$0.04 /cm^2; lids area $= 2\pi r^2$ at \$0.06 /cm^2

$$C(r) = 0.04(2\pi rh) + 0.06(2\pi r^2) = \frac{96}{r} + 0.12\pi r^2 \ \ (\$)$$

(b) Stationary point

$$C'(r) = -\frac{96}{r^2} + 0.24\pi r = 0 \quad \Longrightarrow \quad 0.24\pi r^3 = 96 \Longrightarrow r^3 = \frac{96}{0.24\pi} \approx 0.60/\pi$$

(c) Optimal dimensions and cost

$r \approx (0.60/\pi)^{1/3} \approx 4.1\,\mathrm{cm}$

$h = 1200/(\pi r^2) \approx 22.7\,\text{cm}$
Minimum cost $C \approx \$18.24$