

Graph Transformations – Answer Sheet

- Q1** Right 5, reflect in the x -axis, then down 2
- Q2** Vertical stretch $k = 4$, shift right 2
- Q3** (a) Right 6 and up 3 (b) Reflection in the x -axis
- Q4** Amplitude 3; period π ; vertical shift -1
- Q5** No – it is a horizontal shift left 4 and a vertical shift down 7
- Q6** $y = -2x^3$
- Q7** $y = \frac{1}{x+3} + 5$
- Q8** $y = -(x-1)^2 + 4$
- Q9** $(x+5)^2 + (y-2)^2 = 16$
- Q10** Vertex $(3, -2)$; x -intercepts $(1, 0)$ and $(5, 0)$; y -intercept $(0, -1)$
- Q11** Domain $[0, \infty)$; range $(-\infty, 0]$; decreasing curve through $(0, 0)$
- Q12** One cycle on $0 \leq x \leq 360^\circ$; maximum $(0, 2)$, minima at $x = \pm 90^\circ$
- Q13** $y = -3x^2 - 5$
- Q14** Order A gives $y = \sqrt{\frac{x}{2}} - 3$, order B gives $y = \sqrt{2x - 6} - 3$ – graphs differ
- Q15** $f(x) = \begin{cases} 2x + 1, & x < -1 \\ 0, & -1 \leq x < 3 \\ x - 4, & x \geq 3 \end{cases}$
- Q16** Horizontal steps of height 1 from $y = -2$ up to $y = 2$
- Q17** $f^{-1}(x) = \frac{x+4}{3}$ (reflection in $y = x$)
- Q18** The parabola fails the horizontal-line test unless the domain is restricted, so its inverse is not a function
- Q19** (a) Image of $(2, -3)$ is $(2, 3)$ (b) Pre-image of $(-1, 5)$ is $(-1, -5)$
- Q20** $y = \frac{1}{x-4} - 2$
- Q21** $y = 4 \sin\left(\frac{\pi}{60}(x-15)\right) + 3$
- Q22** Reflect in x -axis; vertical shift up 1; horizontal shrink by factor 2

Challenge – Order Matters

Base curve: $y = \frac{1}{x}$

Transformations to apply

- Horizontal translation 6 units right: $x \mapsto x - 6$
- Reflection in the y -axis: $x \mapsto -x$

(a) Translate first, then reflect

$$y = \frac{1}{x-6} \longrightarrow y = \frac{1}{-x-6} = -\frac{1}{x+6}$$

Final equation: $\boxed{y = -\frac{1}{x+6}}$ Vertical asymptote $x = -6$.

(b) Reflect first, then translate

$$y = -\frac{1}{x} \longrightarrow y = -\frac{1}{x-6}$$

Final equation: $\boxed{y = -\frac{1}{x-6}}$ Vertical asymptote $x = +6$.

(c) Why the graphs are different

The two images look similar but have different vertical asymptotes: $x = -6$ versus $x = +6$. Since an asymptote is a defining geometric feature, the graphs cannot coincide. Algebraically,

$$-\frac{1}{x+6} \neq -\frac{1}{x-6} \quad (\forall x \neq \pm 6).$$

Thus horizontal translations and y -axis reflections do *not* commute: changing their order changes the final location of the graph in the plane.