Area and Volume – Worked Examples

Key Formulas

- Rectangle $A = lw$; Triangle $A = \frac{1}{2}bh$; Parallelogram $A = bh$; Trapezium $A = \frac{1}{2}(a+b)h$.
- Circle $A = \pi r^2$; circumference $C = 2\pi r$.
- Surface area of right prism $= 2A_{\text{base}} + P_{\text{base}}h$; Cylinder $SA = 2\pi r^2 + 2\pi rh$.
- Volume of prism $V = A_{\text{base}}h$; Cylinder $V = \pi r^2 h$.
- Composite shapes $\rightarrow$ split into simpler regions, add/subtract areas or volumes.
- Unit conversions: 1 cm = 10 mm; 1 $\text{m}^3 = 10^6$ cm^3; 1 L = 1000 cm^3.

Example 1 Rectangle

Length 12 cm, width 7 cm.

$$A = 12 \times 7 = \boxed{84 \text{ cm}^2}$$

Example 2 Triangle

Base 9 m, height 6 m.

$$A = \tfrac{1}{2}(9)(6) = \boxed{27 \text{ m}^2}$$

Example 3 Trapezium

Parallel sides 5 cm, 11 cm; height 4 cm.

$$A = \tfrac{1}{2}(5+11)(4) = \boxed{32 \text{ cm}^2}$$

Example 4 Circle

Radius 3.5 cm.

$$A = \pi r^2 = 12.25\pi \approx \boxed{38.5 \text{ cm}^2}$$

Example 5 Surface Area – Closed Cylinder

Radius 4 cm; height 10 cm.

$$SA = 2\pi(4)^2 + 2\pi(4)(10) = 32\pi + 80\pi = 112\pi \approx \boxed{352 \text{ cm}^2}$$

Example 6 Volume – Rectangular Prism

3 m × 2.5 m × 1.2 m.

$$V = 3 \times 2.5 \times 1.2 = \boxed{9.0 \text{ m}^3}$$

Example 7 Volume – Cylinder

Radius 1.2 m, height 2.8 m.

$$V = \pi(1.2)^2(2.8) = 4.03\pi \approx \boxed{12.7 \text{ m}^3}$$

Example 8 Composite Area (L-shape)

Two rectangles: 6 m × 4 m and 3 m × 2 m.

$$A = 24 + 6 = \boxed{30 \text{ m}^2}$$

Example 9 Composite Solid

Right prism 8 cm × 5 cm × 3 cm with half-cylinder (radius 2.5 cm, length 8 cm) on top.

$$V_{\text{prism}} = 120 \text{ cm}^3,\ V_{\text{half-cyl}} = \tfrac{1}{2}\pi(2.5)^2(8) = 25\pi \approx 78.5$$

Total = $\boxed{198.5 \text{ cm}^3}$

Example 10 Area from Scale Drawing

On a map (scale 1 : 200), a park measures 9 cm × 6 cm.

$$\text{Actual } A = (9 \times 200)\,(6 \times 200) = 1800 \times 1200 = 2.16 \times 10^6 \text{ cm}^2 = \boxed{216 \text{ m}^2}$$