Probability & Data – Worked Examples

Key Facts / Formulas

- **Mean**: $\bar{x} = \dfrac{\sum x}{n}$; **median**: middle ordered value; **mode**: most frequent.
- **Sample variance**: $s^2 = \dfrac{\sum(x-\bar{x})^2}{n-1}$; **standard deviation**: $s = \sqrt{s^2}$.
- **Five–number summary**: $\big(\text{min}, Q_1, \text{median}, Q_3, \text{max}\big)$; $\text{IQR} = Q_3 - Q_1$.
- **Relative frequency**: $P(E) \approx \dfrac{\text{frequency of } E}{\text{total trials}}$.
- **Probability rules**: $0 \le P(E) \le 1$; $P(E^c) = 1 - P(E)$; for independent A, B: $P(A \cap B) = P(A)P(B)$.
- **Two–way table**: conditional probability $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.
- **Expected value**: $E(X) = \sum x\, P(X = x)$ for a discrete variable X.
- **Binomial model**: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$; $\mu = np$, $\sigma = \sqrt{np(1-p)}$.

Example 1 Mean, median, mode

Data set (scores): $12, 15, 18, 15, 22, 15, 17, 20$.

$\bar{x} = \dfrac{134}{8} = 16.8$. Ordered: $12, 15, 15, 15, 17, 18, 20, 22$; median $= (15+17)/2 = 16$. Mode $= 15$.

$$\boxed{\bar{x} = 16.8,\ \text{median } 16,\ \text{mode } 15}$$

Example 2 Standard deviation

Using data from Example1.

$$\sum(x-\bar{x})^2 = 46.4; \quad s = \sqrt{\tfrac{46.4}{7}} = 2.57$$

$$\boxed{s \approx 2.6}$$

Example 3 Five–number summary

Weights (kg): $48, 55, 61, 62, 65, 67, 71, 73, 74, 78$.

$\min 48$, $Q_1 = 59$, median $= 64$, $Q_3 = 72$, $\max 78$. $\text{IQR} = 13$.

$$\boxed{(48, 59, 64, 72, 78)}$$

Example 4 Relative frequency estimate

A spinner landed on blue 92 times in 400 spins. Estimate $P(\text{blue})$.

$P \approx 92/400 = 0.23$.

$$\boxed{0.23}$$

Example 5 Complementary probability

Bag: 5 red, 7 green, 4 blue marbles. Draw one at random. $P(\text{not green}) = 1 - P(\text{green}) = 1 - \frac{7}{16} = 0.5625$.

$$\boxed{0.563}$$

Example 6 Conditional probability from two–way table

	Likes Maths	Dislikes Maths	Total
Female	28	12	40
Male	22	18	40
Total	50	30	80

Pick a student who likes maths. $P(\text{female} \mid \text{likes}) = 28/50 = 0.56$.

$$\boxed{0.56}$$

Example 7 Expected value of a game

Fair die: win \$6 for a 6, \$2 for a 5, pay \$1 otherwise.

$E = 6(\frac{1}{6}) + 2(\frac{1}{6}) - 1(\frac{4}{6}) = 1 + 0.333 - 0.667 = 0.666$.

$$\boxed{\$0.67 \text{ gain per roll}}$$

Example 8 Binomial probability

Flip a fair coin 8 times. Probability of exactly 6 heads:

$$P = \binom{8}{6}(0.5)^8 = 28/256 = 0.109$$

$$\boxed{0.109}$$

Example 9 Mean and SD of binomial

For Example8: $\mu = 8(0.5) = 4$, $\sigma = \sqrt{8(0.5)(0.5)} = 1.414$.

$$\boxed{\mu = 4,\ \sigma \approx 1.41}$$

Example 10 Using binomial to approximate expectation

Light-bulb failure probability 0.08. Box of 60 bulbs. Expected failures $\mu = np = 4.8$; SD $\sigma = \sqrt{60(0.08)(0.92)} = 2.1$. Probability at most 3 failures: $P(X \leq 3) = \sum_{r=0}^{3} \binom{60}{r} 0.08^r 0.92^{60-r} = 0.35$ (CAS).

$$\boxed{P(X \leq 3) \approx 0.35}$$