

Working with Functions – Question Set

Index Laws & Simplification

Q1 (a) Simplify $5a^{-3}b^2 \times 2a^4b^{-5}$

(b) Express $\frac{x^7}{x^{-2}}$ with positive indices

Q2 Write as a single power of p : $p^4 \div p^{\frac{3}{2}}$.

Q3 Simplify $\frac{9m^2 - 25}{3m - 5}$ where possible.

Q4 Rationalise and simplify $\frac{6}{\sqrt{3}}$.

Functions & Relations

Q5 State whether each set of ordered pairs is a function: (a) $\{(1, 4), (2, 5), (3, 6)\}$

Q6 Does the graph of $|y| = x$ represent y as a function of x ? Explain.

Q7 Write the domain and range of $f(x) = \frac{1}{x - 4}$.

Even, Odd & Symmetry

Q8 Determine whether $f(x) = x^4 - 7x^2$ is even, odd, or neither.

Q9 Show that $g(x) = \frac{x}{x^2 + 1}$ is an odd function.

Composite & Inverse Functions

Q10 Given $f(x) = 4 - x^2$ and $g(x) = \sqrt{x}$ (domain $x \geq 0$):

(a) Find $(g \circ f)(2)$.

(b) State the domain of $g \circ f$.

Q11 Let $h(x) = 3x + 7$. Find $h^{-1}(x)$ and check that $h(h^{-1}(x)) = x$.

Quadratics & Discriminant

Q12 For $y = 2x^2 + kx + 8$, determine k if the curve just touches the x -axis.

Q13 How many real solutions has $t^2 + 5t + 9 = 0$? Justify.

Q14 Solve $3x^2 - 13x + 4 = 0$.

Graphing & Piecewise

Q15 Sketch $y = (x - 2)^2 - 4$, marking vertex, axis of symmetry and intercepts.

Q16 The function $p(x)$ is defined

$$p(x) = \begin{cases} x + 4, & x < -2 \\ x^2, & -2 \leq x < 3 \\ 8 - x, & x \geq 3 \end{cases}$$

(a) Evaluate $p(-3)$, $p(0)$, $p(4)$.

(b) State the range of p .

Direct & Inverse Variation

Q17 If y varies directly as x^3 and $y = 40$ when $x = 2$, find y when $x = 3$.

Q18 The pressure P of a gas varies inversely with volume V . When $P = 250\text{kPa}$, $V = 3.2\text{L}$. Find P when $V = 4.8\text{L}$.

Circles & Distance

Q19 Find the centre and radius of $(x - 5)^2 + y^2 = 20$.

Q20 Does point $(9, 3)$ lie inside, on, or outside this circle?

Q21 Write the equation of the circle with diameter endpoints $(2, -1)$ and $(8, 5)$.

Mixed Practice

Q22 (a) Simplify $\frac{5}{x} - \frac{3}{x+2}$

(b) Solve for x : $4^x = 64$

Q23 A parabola $y = ax^2 + bx + c$ passes through $(1, 6)$, $(2, 9)$, $(4, 25)$. Find a , b , c .

Q24 Explain why the relation $x^2 = 4y$ fails the vertical-line test after rotating the axes 90° .

Q25 A function f satisfies $f(1) = 3$ and $f^{-1}(3) = 1$. Explain what this tells you about the point on the graph of f^{-1} .

Challenge

Q26 Modelling an Epidemic The number of active cases C fits $C(t) = \frac{1200}{1 + ae^{-0.45t}}$ where t is days. Data show $C(0) = 150$.

- (a) Find the constant a .
- (b) Determine the limiting number of active cases.
- (c) How many days until active cases reach 900?
- (d) Comment on whether the model is a function and justify its domain.