

## Working with Functions – Question Set

### Index Laws & Simplification

**Q1** (a) Simplify  $5a^{-3}b^2 \times 2a^4b^{-5}$

(b) Express  $\frac{x^7}{x^{-2}}$  with positive indices

**Q2** Write as a single power of  $p$ :  $p^4 \div p^{\frac{3}{2}}$ .

**Q3** Simplify  $\frac{9m^2 - 25}{3m - 5}$  where possible.

**Q4** Rationalise and simplify  $\frac{6}{\sqrt{3}}$ .

### Functions & Relations

**Q5** State whether each set of ordered pairs is a function: (a)  $\{(1, 4), (2, 5), (3, 6)\}$

**Q6** Does the graph of  $|y| = x$  represent  $y$  as a function of  $x$ ? Explain.

**Q7** Write the domain and range of  $f(x) = \frac{1}{x-4}$ .

### Even, Odd & Symmetry

**Q8** Determine whether  $f(x) = x^4 - 7x^2$  is even, odd, or neither.

**Q9** Show that  $g(x) = \frac{x}{x^2 + 1}$  is an odd function.

### Composite & Inverse Functions

**Q10** Given  $f(x) = 4 - x^2$  and  $g(x) = \sqrt{x}$  (domain  $x \geq 0$ ):

(a) Find  $(g \circ f)(2)$ .

(b) State the domain of  $g \circ f$ .

**Q11** Let  $h(x) = 3x + 7$ . Find  $h^{-1}(x)$  and check that  $h(h^{-1}(x)) = x$ .

## Quadratics & Discriminant

**Q12** For  $y = 2x^2 + kx + 8$ , determine  $k$  if the curve just touches the  $x$ -axis.

**Q13** How many real solutions has  $t^2 + 5t + 9 = 0$ ? Justify.

**Q14** Solve  $3x^2 - 13x + 4 = 0$ .

# Graphing & Piecewise

**Q15** Sketch  $y = (x - 2)^2 - 4$ , marking vertex, axis of symmetry and intercepts.

**Q16** The function  $p(x)$  is defined

$$p(x) = \begin{cases} x + 4, & x < -2 \\ x^2, & -2 \leq x < 3 \\ 8 - x, & x \geq 3 \end{cases}$$

(a) Evaluate  $p(-3)$ ,  $p(0)$ ,  $p(4)$ .  
 (b) State the range of  $p$ .

## Direct & Inverse Variation

**Q17** If  $y$  varies directly as  $x^3$  and  $y = 40$  when  $x = 2$ , find  $y$  when  $x = 3$ .

**Q18** The pressure  $P$  of a gas varies inversely with volume  $V$ . When  $P = 250\text{kPa}$ ,  $V = 3.2\text{L}$ . Find  $P$  when  $V = 4.8\text{L}$ .

## Circles & Distance

**Q19** Find the centre and radius of  $(x - 5)^2 + y^2 = 20$ .

**Q20** Does point  $(9, 3)$  lie inside, on, or outside this circle?

**Q21** Write the equation of the circle with diameter endpoints  $(2, -1)$  and  $(8, 5)$ .

## Mixed Practice

**Q22** (a) Simplify  $\frac{5}{x} - \frac{3}{x+2}$  (b) Solve for  $x$ :  $4^x = 64$

**Q23** A parabola  $y = ax^2 + bx + c$  passes through  $(1, 6)$ ,  $(2, 9)$ ,  $(4, 25)$ . Find  $a, b, c$ .

**Q24** Explain why the relation  $x^2 = 4y$  fails the vertical-line test after rotating the axes  $90^\circ$ .

**Q25** A function  $f$  satisfies  $f(1) = 3$  and  $f^{-1}(3) = 1$ . Explain what this tells you about the point on the graph of  $f^{-1}$ .

## Challenge

**Q26 Modelling an Epidemic** The number of active cases  $C$  fits  $C(t) = \frac{1200}{1 + ae^{-0.45t}}$  where  $t$  is days. Data show  $C(0) = 150$ .

- (a) Find the constant  $a$ .
- (b) Determine the limiting number of active cases.
- (c) How many days until active cases reach 900?
- (d) Comment on whether the model is a function and justify its domain.