

Properties of Geometrical Figures – Worked Examples

Key Theorems & Facts

- **Interior-angle sum of an n -gon:** $(n - 2) \times 180^\circ$.
- **Exterior-angle sum:** always 360° for any convex polygon.
- **Special quadrilaterals** Parallelogram: opp. sides \parallel and equal; opp. angles equal; diagonals bisect. Rectangle: parallelogram + all angles 90° . Rhombus: parallelogram + all sides equal; diagonals bisect angles. Kite: two pairs adj. equal sides; one pair opp. angles equal; diagonals .
- **Congruence tests:** SSS, SAS, ASA/AAS, RHS.
- **Similarity tests:** AAA, SAS (ratio), SSS (ratio).
- **Mid-segment (triangle):** segment joining mid-points of two sides is \parallel third side and half its length.
- **Parallel-line proportion:** if $DE \parallel BC$ in $\triangle ABC$, then $\frac{AD}{DB} = \frac{AE}{EC}$.
- **Exterior angle of triangle** equals sum of opposite interior angles.
- **Angle sum in quadrilateral:** 360° .

Example 1 Regular Decagon Interior Angle

$$(n - 2)180^\circ = 8 \times 180^\circ = 1440^\circ; \text{ each} = \frac{1440^\circ}{10} = \boxed{144^\circ}$$

Example 2 Exterior Angle of a Regular 18-gon

$$\text{Exterior} = \frac{360^\circ}{18} = \boxed{20^\circ}$$

Example 3 Parallelogram Angles

In $ABCD$, $\angle A = 68^\circ$. Opposite $\angle C = 68^\circ$; adjacent $\angle B = \angle D = 112^\circ$ (supplementary).

$$\boxed{\angle B = \angle D = 112^\circ}, \boxed{\angle C = 68^\circ}$$

Example 4 Rhombus Diagonal

Rhombus with side 10 cm and one diagonal 16 cm. Diagonals are perpendicular bisectors, so half-diagonals and side form right . Half unknown diagonal x :

$$x^2 + 8^2 = 10^2 \Rightarrow x = 6 \text{ (cm)}, \text{ full diag} = 12 \text{ cm}$$

$$\boxed{12 \text{ cm}}$$

Example 5 Congruence (ASA)

$\triangle PQR$ and $\triangle STU$ have $PQ = ST$, $\angle P = \angle S$, $\angle Q = \angle T$. By ASA \rightarrow triangles congruent $PR = SU$.

$$\boxed{PR = SU}$$

Example 6 Similarity Ratio

$\triangle ABC \sim \triangle DEF$ with scale factor $k = \frac{AB}{DE} = \frac{15}{9} = \frac{5}{3}$. If $EF = 6$ cm find BC :

$$BC = k \cdot EF = \frac{5}{3} \times 6 = \boxed{10 \text{ cm}}$$

Example 7 Mid-segment Length

In $\triangle XYZ$ mid-points M, N on XY, XZ . If $YZ = 14$ cm then $MN = \frac{1}{2} \times 14 = \boxed{7 \text{ cm}}$.

Example 8 Parallel-line Proportion

In $\triangle ABC$ with $DE \parallel BC$, $AD = 3$ cm, $DB = 5$ cm, $AE = 4.2$ cm.

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{5} = \frac{4.2}{EC} \Rightarrow EC = \boxed{7 \text{ cm}}$$

Example 9 Exterior Angle of Triangle

At vertex C exterior angle $= 128^\circ$. Opposite interior angles A, B are 42° and x :

$$x + 42^\circ = 128^\circ \Rightarrow x = \boxed{86^\circ}$$

Example 10 Angles in a Kite

In kite $ABCD$ with $AB = AD$ and $CB = CD$, $\angle BAD = 48^\circ$. Opposite equal $\angle BCD = 48^\circ$. Using quad sum, other angles each $\frac{1}{2}(360^\circ - 96^\circ) = \boxed{132^\circ}$.