Financial Mathematics – Worked Examples

Key Facts / Formulas

- Simple interest: $I = Prt$, amount $A = P(1 + rt)$ where r is per year and t in years.
- Compound interest m times per year: $A = P\left(1 + \frac{r}{m}\right)^{mt}$.
- Continuous compounding: $A = Pe^{rt}$.
- Effective annual rate (EAR) for nominal r compounded m times: $i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^{m} - 1$.
- Inflation/real growth: real rate $r_{\text{real}} \approx \dfrac{1+i}{1+\pi} - 1$, where i is nominal investment rate and π the inflation rate.
- Present value (discounting): $PV = \dfrac{A}{(1+i)^n}$ for a single future payment A after n periods at rate i per period.
- Level annuity immediate (payments at period ends):
  Future value $FV = R\,\dfrac{(1+i)^n - 1}{i}$, Present value $PV = R\,\dfrac{1-(1+i)^{-n}}{i}$.
- Reducing loan with level repayments R each period at rate i: same PV formula with $PV =$ initial loan L, so $R = L\,\dfrac{i}{1-(1+i)^{-n}}$.
- Declining balance depreciation: $V_t = V_0(1-d)^t$; straight line depreciation: $V_t = V_0 - t \cdot \dfrac{V_0 - S}{n}$.

Example 1 Simple interest over a short term

A term deposit of \$7 500 earns simple interest at 4.2 % p.a. for 18 months. Find the interest earned.

Convert time to years: $t = 1.5$. Interest $I = Prt = 7500(0.042)(1.5) = 472.5$.

$$\boxed{\$472.50}$$

Example 2 Compound interest quarterly

\$12 000 is invested at 5.6 % p.a. compounded quarterly for 4 years. Find the future value.

$m = 4$, $r = 0.056$, $t = 4$.

$$A = P\left(1 + \frac{r}{m}\right)^{mt} = 12000\left(1 + \frac{0.056}{4}\right)^{16} = 12000(1.014)^{16}.$$

$(1.014)^{16} = 1.2373$ (to 4 d.p.). So $A \approx 12000(1.2373) = 14\,847.6$.

$$\boxed{\$14\,847.60 \text{ (approx.)}}$$

Example 3 Continuous compounding comparison

How long will it take an investment to double if it earns 6.0 % p.a. compounded continuously?

$$A = Pe^{rt} \text{ with } A = 2P \Rightarrow 2 = e^{0.06t} \Rightarrow t = \frac{\ln 2}{0.06} = 11.552.$$

$$\boxed{11.55 \text{ years (to 2 d.p.)}}$$

Example 4 Effective annual rate from a nominal rate

A credit card advertises 18.0 % p.a. compounded monthly. Find the effective annual rate.

$i_{\text{eff}} = (1 + \frac{0.18}{12})^{12} - 1 = (1.015)^{12} - 1.$
$(1.015)^{12} = 1.1956$ (to 4 d.p.), so $i_{\text{eff}} = 0.1956 = 19.56$ %.

$$\boxed{19.56 \text{ \% p.a.}}$$

Example 5 Real rate allowing for inflation

An investment fund returns 7.2 % p.a. (effective). Inflation averages 2.6 % p.a. Find the real rate.

$$r_{\text{real}} = \frac{1 + 0.072}{1 + 0.026} - 1 = \frac{1.072}{1.026} - 1 = 0.0448.$$

$$\boxed{4.48 \text{ \% p.a. (real)}}$$

Example 6 Present value of a future goal

You need \$25 000 in 5 years. If you can earn 5.1 % p.a. compounded annually, how much must be invested now?

$PV = \dfrac{A}{(1+i)^n} = \dfrac{25000}{(1.051)^5}$. $(1.051)^5 = 1.2830$.
$PV \approx 25000/1.2830 = 19\,486.7$.

$$\boxed{\$19\,486.70 \text{ (approx.)}}$$

Example 7 Annuity future value (regular savings)

You deposit \$350 at the end of each month into an account paying 4.8 % p.a. compounded monthly. What is the balance after 6 years?

Monthly rate $i = \frac{0.048}{12} = 0.004$, periods $n = 72$.

$$FV = R\frac{(1+i)^n - 1}{i} = 350\frac{(1.004)^{72} - 1}{0.004}.$$

$(1.004)^{72} = 1.3319$; fraction $= \dfrac{0.3319}{0.004} = 82.9875$.
$FV \approx 350(82.9875) = 29\,045.6$.

$$\boxed{\$29\,045.60 \text{ (approx.)}}$$

Example 8 Level repayment on a reducing loan

A car loan of \$28 000 is to be repaid monthly over 5 years at 6.6 % p.a. compounding monthly. Find the monthly repayment.

$i = \frac{0.066}{12} = 0.0055$, $n = 60$, $L = 28000$.
$R = L\dfrac{i}{1-(1+i)^{-n}} = 28000\dfrac{0.0055}{1-(1.0055)^{-60}}$.
$(1.0055)^{-60} = 0.7288$; denominator $= 1 - 0.7288 = 0.2712$.
$R \approx 28000 \times \dfrac{0.0055}{0.2712} = 28000 \times 0.020284 = 568.0$.

$$\boxed{\$568.00 \text{ per month (approx.)}}$$

Example 9 Loan principal remaining after partial term

Using Example 8's loan, what principal remains after 2 years of repayments?

Outstanding balance equals PV of remaining payments. After 24 payments, $n_{\text{rem}} = 36$.
$B = R\dfrac{1-(1+i)^{-n_{\text{rem}}}}{i} = 568.00\dfrac{1-(1.0055)^{-36}}{0.0055}$.
$(1.0055)^{-36} = 0.8204$; numerator $= 1 - 0.8204 = 0.1796$; fraction $= 32.6545$.
$B \approx 568.00(32.6545) = 18\,558.6$.

$$\boxed{\$18\,558.60 \text{ (approx.)}}$$

Example 10 Declining balance depreciation

Equipment costing \$62 000 depreciates at 18 % p.a. reducing balance. What is its value after 5 years?

$V = 62000(1-0.18)^5 = 62000(0.82)^5$. $(0.82)^5 = 0.3707$. $V \approx 62000(0.3707) = 22\,983.4$.

$$\boxed{\$22\,983.40 \text{ (approx.)}}$$