

Circle Geometry – Worked Examples

Key Theorems & Facts

- Angle subtended by an arc at the centre = $2 \times$ the angle at any point on the circumference.
- Angles in the same segment (standing on the same chord) are equal.
- Angle in a semicircle is 90° .
- Opposite angles of a cyclic quadrilateral sum to 180° .
- The perpendicular from the centre to a chord bisects the chord.
- Tangent–secant theorem (alternate-segment): angle between tangent and chord equals angle in the alternate segment.
- Power of a point: $EA \times EB = EC \times ED$ for two chords through one point E .
- Two tangents from the same external point are equal in length.
- Arc length $s = r\theta$ (in radians); sector area $A = \frac{1}{2}r^2\theta$.

Example 1 Angle in a Semicircle

In circle O , diameter AB . C lies on the circumference. $\angle ACB = 90^\circ$ by “angle in a semicircle” theorem.

$$\boxed{90^\circ}$$

Example 2 Centre vs Circumference

Arc PQ subtends 60° at the centre. Angle PRQ on the circumference standing on same arc = 30° .

$$\boxed{30^\circ}$$

Example 3 Equal Angles in Same Segment

Chord XY subtends $\angle XZY = 42^\circ$. Any other point W on the same segment gives $\angle XWY = 42^\circ$.

$$\boxed{42^\circ}$$

Example 4 Cyclic Quadrilateral

In cyclic quad $ABCD$, $\angle A = 78^\circ$. Opposite angle $\angle C = 180^\circ - 78^\circ = \boxed{102^\circ}$.

Example 5 Tangent–Chord Theorem

At point T on circle, tangent TM and chord TP form 40° . Angle in opposite segment $\angle TRP = 40^\circ$.

$$\boxed{40^\circ}$$

Example 6 Chord Bisected by Perpendicular

Chord length 12 cm, distance from centre to chord 5 cm. Radius r found by right-triangle: $r^2 = 5^2 + 6^2 = 61 \Rightarrow r = \boxed{7.81 \text{ cm}}$.

Example 7 Power of a Point

Two chords through point E outside circle: $EA = 4 \text{ cm}$, $EB = 10 \text{ cm}$, $EC = 3 \text{ cm}$. Find ED .

$$EA \cdot EB = EC \cdot ED \Rightarrow 4 \times 10 = 3 \times ED \Rightarrow ED = \boxed{13.\bar{3} \text{ cm}}$$

Example 8 Equal Tangents

From P two tangents touch circle at Q, R . If $PQ = 11 \text{ cm}$, then $PR = \boxed{11 \text{ cm}}$.

Example 9 Arc Length

Circle radius 7 cm; central angle $135^\circ = \frac{3\pi}{4} \text{ rad}$.

$$s = r\theta = 7 \times \frac{3\pi}{4} = \frac{21\pi}{4} \approx \boxed{16.5 \text{ cm}}$$

Example 10 Sector Area

Same circle and angle: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(49)\frac{3\pi}{4} = \frac{147\pi}{8} \approx \boxed{57.8 \text{ cm}^2}$