Trigonometry – Answer Sheet

Q1 (a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{\sqrt{3}}{2}$

Q2 (a) $210^\circ = \dfrac{7\pi}{6}$ rad (b) -105°

Q3 (a) $\sin\left(\frac{11\pi}{6}\right) = -\dfrac{1}{2}$ (b) $\cos 495^\circ = \cos 135^\circ = -\dfrac{\sqrt{2}}{2}$

Q4 $\theta = \cos^{-1}\left(\frac{1.4}{4.8}\right) = 73^\circ$

Q5 Horizontal distance $= 38/\tan 27^\circ \approx 75$ m

Q6 $a = \dfrac{8.2 \sin 42^\circ}{\sin 67^\circ} \approx 6.3$ cm

Q7 Largest angle opposite 6.4 cm: $\cos\theta = \dfrac{5.1^2 + 4.7^2 - 6.4^2}{2(5.1)(4.7)} = -0.136\ \theta \approx 98^\circ$

Q8 $\theta = \dfrac{2A}{r^2} = \dfrac{2(72)}{9.5^2} = 1.59$ rad

Q9 Arc $= \dfrac{420^\circ}{360^\circ}(2\pi r) = \dfrac{7\pi}{3}(0.32) \approx 2.34$ m

Q10 Expression $= 0$ (same cancellation shown in notes)

Q11 $\dfrac{1-\cos^2\theta}{\sin\theta} = \dfrac{\sin^2\theta}{\sin\theta} = \sin\theta$

Q12 Since tan has period π: $\tan(\theta+\pi) = \tan\theta$ (proof via sine/cosine)

Q13 $\sin x = \dfrac{\sqrt{3}}{2} \Rightarrow x = 60^\circ, 120^\circ$

Q14 $\cos^2\theta = \dfrac{3}{4} \Rightarrow \cos\theta = \pm\dfrac{\sqrt{3}}{2}$. $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

Q15 $\sin 2t = \sin t \Rightarrow 2t = t + k\pi$ or $2t = \pi - t + k\pi$ Solutions $t = 0^\circ,\ 180^\circ,\ 60^\circ,\ 300^\circ$

Q16 Components: First leg $(+5.20, -4.33)$ km, second $(+3.46, -3.46)$ km. Resultant $(8.66, -7.79)$; distance $= 11.7$ km, bearing 222°.

Q17 (a) Cotangent is reciprocal of tangent; using co-angle identity gives result. (b) $\cot 30^\circ = \sqrt{3}$

Q18 Height $= 21 \tan 62^\circ \approx 39$ m

Q19 $A = 3$, $B = \frac{2\pi}{180} = \frac{\pi}{90}$, $C = -\frac{\pi}{6}$, $D = 2$ So: $y = 3\sin\left(\frac{\pi}{90}x - \frac{\pi}{6}\right) + 2$

Challenge — Worked Solution

Diagram – two right triangles share cliff height h.

Let d be width of gorge AB. From the first position:

$$\tan 18^\circ = \frac{h}{d} \quad \Longrightarrow \quad h = d\tan 18^\circ$$

From the second position ($d - 75$ metres away):

$$\tan 25^\circ = \frac{h}{d-75}$$

Equate h:

$$d\tan 18^\circ = (d-75)\tan 25^\circ \Longrightarrow d = \frac{75\tan 25^\circ}{\tan 25^\circ - \tan 18^\circ} = 183 \text{ m (nearest m)}$$

$$h = d\tan 18^\circ = 183\tan 18^\circ \approx 59 \text{ m}$$

$$\boxed{\text{Cliff 59 m high; gorge width 183 m}}$$