

# Exponential & Logarithmic Functions – Answer Sheet

**Q1** (a)  $x = \log_3 17$  (b)  $2^y = 9$

**Q2**  $\log_5 42 \approx 2.322$   $\ln 0.35 \approx -1.050$

**Q3** (a) 3 (b) 2 (c)  $-4$

**Q4**  $\log_{10} 6 < \log_5 6 < \log_2 6$

**Q5**  $\log_a 12 + 3 \log_a x + \frac{1}{2} \log_a y$

**Q6**  $\frac{1}{3}$

**Q7** Using laws:  $\log_b(pq/r)$

**Q8** Exact  $x = \log_4 96$ ;  $x \approx 3.29$

**Q9**  $3x - 5 = e^2 \Rightarrow x \approx 4.13$

**Q10**  $x = \frac{\ln 2}{2 \ln 2 - \ln 5} \approx -3.11$

**Q11**  $t = \frac{\ln(88/7)}{0.15} \approx 16.9$

**Q12**  $b \approx 1.14$ ;  $N_6 \approx 2.0 \times 10^5$

**Q13** (a)  $b = 0.88$  (b) half-life  $t = \frac{\ln 0.5}{\ln 0.88} \approx 325 \text{ min}$

**Q14**  $t = \frac{\ln 2}{\ln 1.0035} \approx 16.5 \text{ y}$

**Q15** Intercept  $(0, 5)$ , HA  $y = 0$ , increasing

**Q16** Axis  $x = 3$ ; vertex  $(3, 5)$

**Q17** VA  $x = -3$ ; HA  $y = -1$ ;  $y$ -int  $5/3$

**Q18** Base  $< 1$  so graph decreases

**Q19** (a)  $E(0) = 140 \text{ W}$  (b)  $t = \frac{\ln(40/140)}{-0.07} \approx 17.9 \text{ h}$

**Q20**  $S_5 = 4200(1 - e^{-1.5}) \approx 3260$ ; limit  $= 4200$

**Q21**  $C = 220\left(\frac{1}{2}\right)^{t/1.5}$ ;  $C_{4h} \approx 34.5 \text{ mg}$

**Q22** (a) shown by change-of-base (b)  $\log_7 27 \approx 1.693$

**Q23** Gradients:  $e^x$  rises faster ( $1 > 0.693$  at  $x = 0$ )

**Q24**  $\frac{d}{dx} \log_3 x = \frac{1}{x \ln 3}$

## Challenge — Worked Solution

**Model**  $P(t) = \frac{80}{1 + 15e^{-0.45t}}$  (thousands)

(a)  $P(0) = \frac{80}{1 + 15} = 5$

(b)  $\lim_{t \rightarrow \infty} P(t) = 80$  (thousand)

(c) Solve  $\frac{80}{1 + 15e^{-0.45t}} = 60$   
 $1 + 15e^{-0.45t} = \frac{4}{3}$   
 $15e^{-0.45t} = \frac{1}{3}$

$$e^{-0.45t} = \frac{1}{45}$$

$$t = \frac{\ln 45}{0.45} \approx 8.46 \text{ y}$$

- (d)  $P'(t) = \frac{80(15)(0.45)e^{-0.45t}}{(1+15e^{-0.45t})^2} > 0$  for all  $t$ ; at  $t = 0$  growth is rapid,  $P' \approx 54$ (thousand  $\text{y}^{-1}$ ); as  $t \rightarrow \infty$ ,  $e^{-0.45t} \rightarrow 0$  so  $P' \rightarrow 0$ : growth slows near the carrying capacity.