Probability & Data – Answer Sheet

Q1 $\bar{x} = \dfrac{157}{8} = 19.6$; ordered data $15, 17, 18, 19, 20, 22, 22, 24$ median $= \dfrac{19+20}{2} = 19.5$; mode $= 22$

Q2 $s = \sqrt{\dfrac{\sum(x-\bar{x})^2}{7}} \approx 3.0$

Q3 Heights (ordered): $162, 165, 168, 169, 170, 171, 174, 175, 177, 182$ min 162, $Q_1 = 168$, median $= 170.5$, $Q_3 = 175$, max 182 IQR $= 7$

Q4 Longer upper tail (182 far above 162) slight positive skew

Q5 Box plot shows median (centre) and IQR/whiskers (spread)

Q6 Cluster near 70 with right-tail most students scored well; few lower marks

Q7 (a) $5/15 = 0.333$ (b) $9/15 = 0.600$

Q8 Remaining probability $= 0.70$ shared equally by 5 faces $P(3) = 0.14$

Q9 (a) $P(A \cap B) = 0.45 \times 0.20 = 0.090$ (b) $P(A \cup B) = 0.45 + 0.20 - 0.090 = 0.560$

Q10 Complement: "no heads" $= \left(\frac{1}{2}\right)^3 = 0.125$

Q11 $P(\text{yellow}) \approx 64/250 = 0.256$

Q12 Expected yellows $= 0.256 \times 750 \approx 192$

Q13 Completed table:

	Owns Car	No Car	Total
Under 25	18	47	65
25+	54	21	75
Total	72	68	140

Q14 Frequencies filled above

Q15 $P(\text{car}) = 72/140 = 0.514$

Q16 $P(\text{under } 25 \mid \text{no car}) = 47/68 = 0.691$

Q17 $E = 10\left(\frac{1}{6}\right) + 4\left(\frac{2}{6}\right) - 2\left(\frac{3}{6}\right) = 2$

Q18 Paying \$1 leaves expected profit \$1 game is favourable

Q19 (a) $\binom{12}{7}0.5^{12} = 792/4096 \approx 0.193$ (b) Heads ≥ 9: probability $= 299/4096 \approx 0.073$

Q20 $P(X \geq 6) = \sum_{r=6}^{8} \binom{8}{r}0.75^r 0.25^{8-r} \approx 0.676$

Q21 $\mu = np = 8(0.75) = 6$; $\sigma = \sqrt{8(0.75)(0.25)} \approx 1.23$

Challenge — Worked Solution

(a) Model $X \sim B(120, 0.90)$, mean $\mu = np = 108$

(b) Probability $X \leq 98$

Using a calculator/CAS:

$$P(X \leq 98) \approx 0.003$$

(c) Interpretation

A result this low (0.3 percent chance) is very unlikely if the true germination rate is 90 percent. Therefore the claim appears *over-stated*; actual rate is probably lower.